How Mathematics Has Helped Shape Civilization

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When did we start?

- Humans didn’t eveolve with a compulsion to count.
- But eventually we did, and after numbers & arithmetic was invented, it paved the way for living in large independent societies.
- We could tax, trade & lend with the help of numbers.
- In fact, the earliest known piece of arithmetic was an accounting table!
- But the first recorded counting is even older, at 20,000 years (maybe even older)! Humans made tally marks.

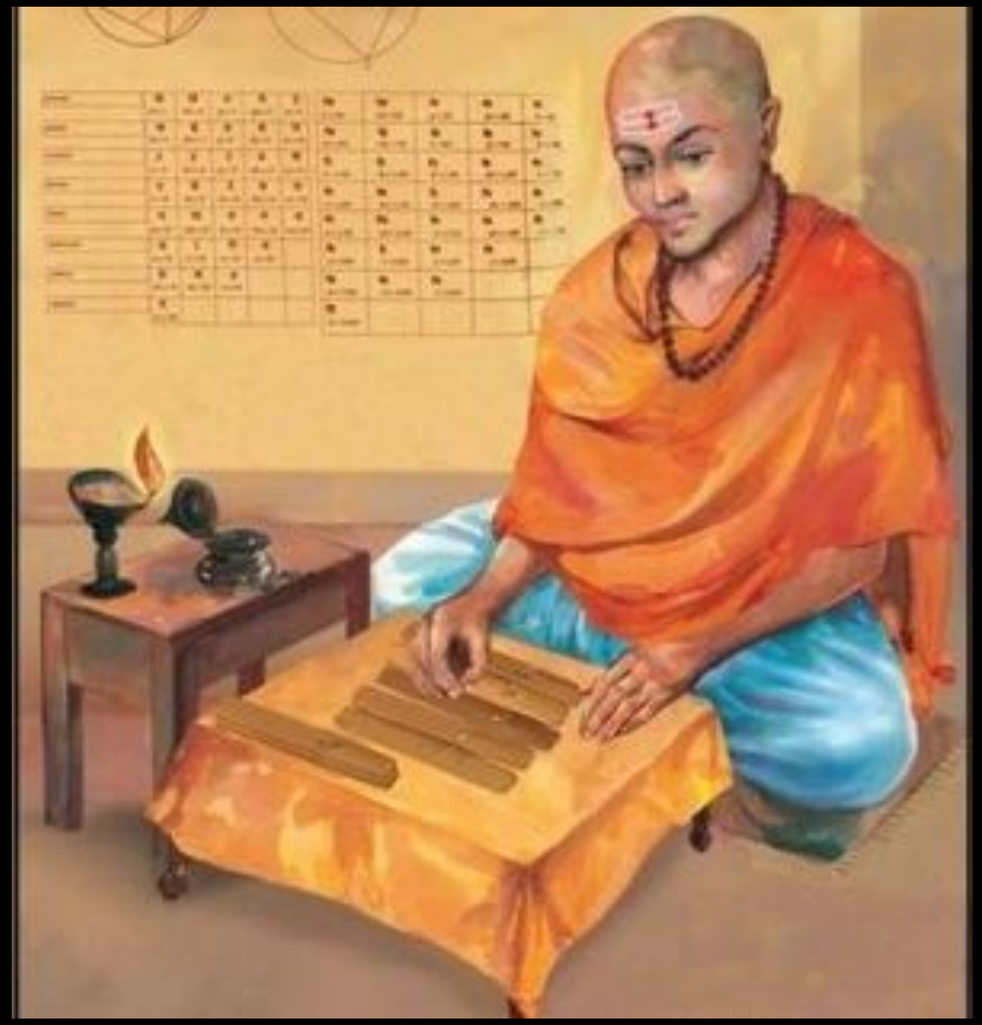

a
1
2
3
4
5
6
7
8
9
b
c
d

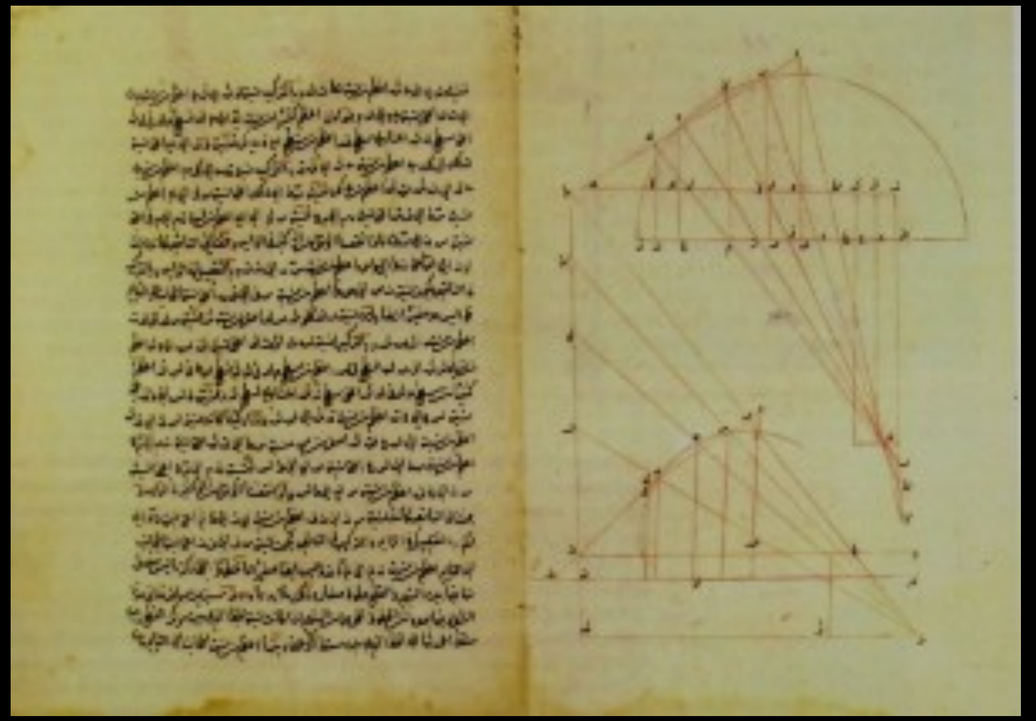
What next?

- Counting started with just fingers, so even with the absence of language we could trade.
- Accounting practises helped sustain businesses.
- After positive numbers, fractions were naturally introduced to naturally divide things: crops, land or even profit from business.

Negativeness

- Negativeness is not easy to imagine, so it was a major leap forward.
- It was started by Indian and Chinese accountants to represent debts around 300 BC to 50 AD.
- Brahmagupta in AD 628 gave the ‘minus times minus is plus’ rule.
- In the west it was very slow because they were focused on Greek math, first steps were by Fibonacci in Liber Abaci (Book of Calculation).



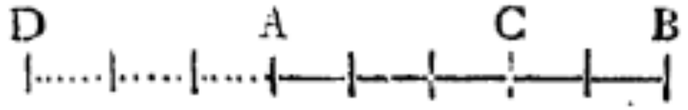


Some European Developments

- Fibonacci gave a series of simultaneous linear equations, whose solution could be negative.
- But things moved very slowly, even in the 17th century Pascal was reluctant to accept negative numbers.
- John Wallis (Savilian Prof. of Math at Oxford) in 1685 published, ‘A Treatise of Algebra’ where he helped visualize negative numbers, by the concept of the number line.
- In physics, Negative numbers eased so much (for physical interpretations such as negative forces, etc).
- It took almost all of human history to come to negative numbers! The rest of the 320 years have had tremendous progress.

Yet is not that Suppoſition (of Negative Quantities,) either Unuſeful or Abſurd; when rightly underſtood. And though, as to the bare Algebraick Notation, it import a Quantity leſs than nothing: Yet, when it comes to a Phyſical Application, it denotes as Real a Quantity as if the Sign were +; but to be interpreted in a contrary ſenſe.

As for inſtance: Suppoſing a man to have advanced or moved forward, (from A to B,) 5 Yards; and then to retreat (from B to C) 2 Yards: If it be asked, how much he had Advanced (upon the whole march) when at C? or how many Yards he is now Forwarder than when he was at A? I find (by Subducting 2 from 5,) that he is Advanced 3 Yards. (Becauſe $+5-2=+3$.)



Zero

- Zero as a placeholder started out in the Babylonian clay tablets.
- Generally accepted oldest known 0 is in the Bakshali manuscript (224-383 AD). This was also a placeholder.
- As a place in the number line, it was by Brahmagupta (628 AD). He gave propositions such as a+0=0, 0+0=0, a x 0=0, 0 x 0=0.
- The west came to know of these developments when Al-Khwarizmi wrote his book (830 AD), and gave it as a tool in algebra.
- But acceptance was gradual, and the earliest official use wa in 1305 in Pisa (again for keeping accounts).
- From this event, western civilization accelerated at a great pace compared to the last 300k years of human history.
- Then came double entry bookkeeping, which revolutionized how business was done. (Specially in Italy which served as sea ports for merchants in the east and the west.)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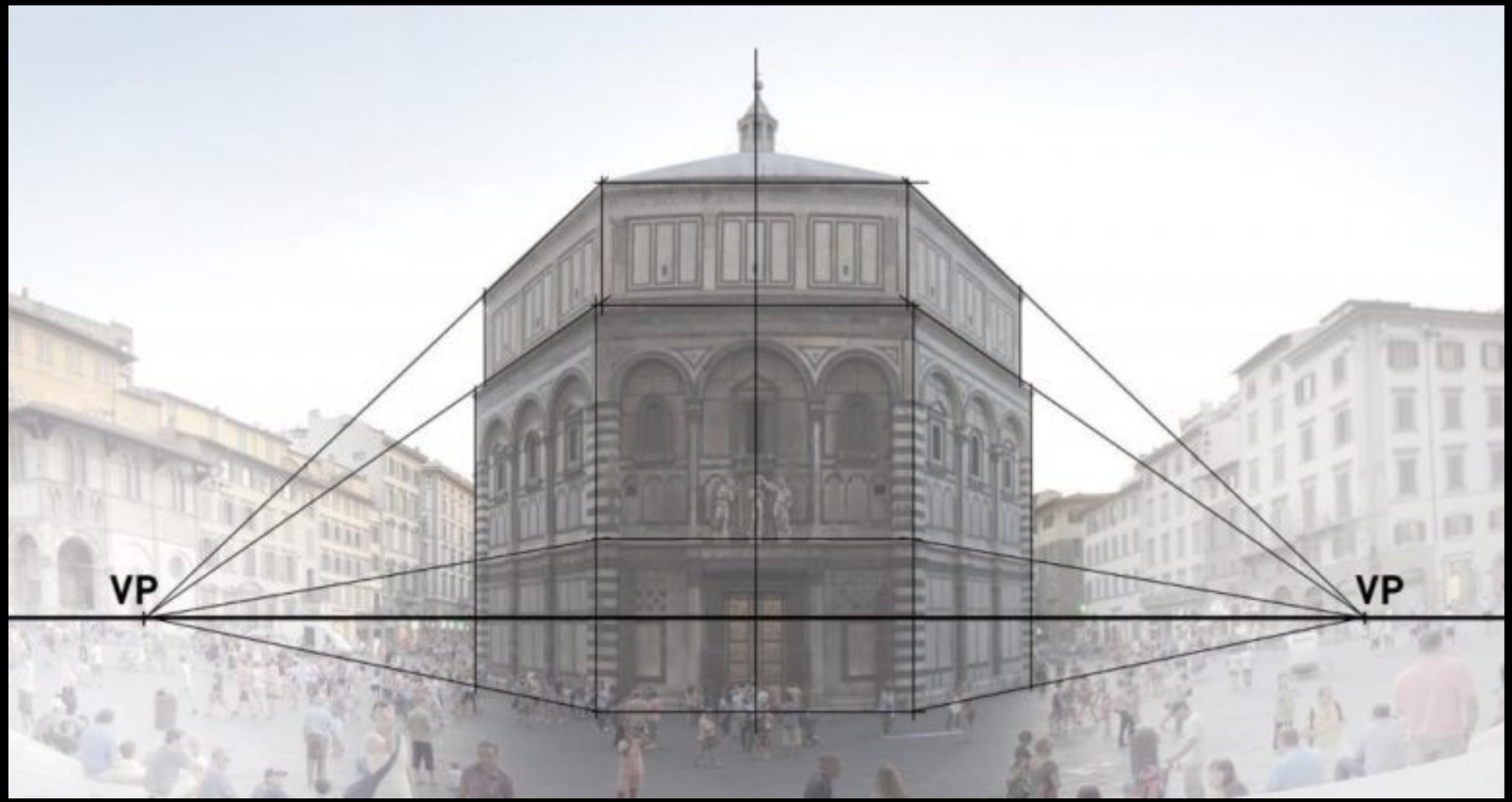
Geometry

- The marvels of mathematics was not only limited to accounting, but also to the renaissance, which occured in the same place as the accounting renaissance (in northern Italy).
- To delve more into this, we now look at geometry, which allowed world exploration via maps, architecture, etc.
- The hidden powers of triangles had been known even to Pythagora (eg. Irrational numbers).
- Geometry was essential for the construction of marvels such as the Egyptian pyramids, Mughals gardens (in more recent times), etc.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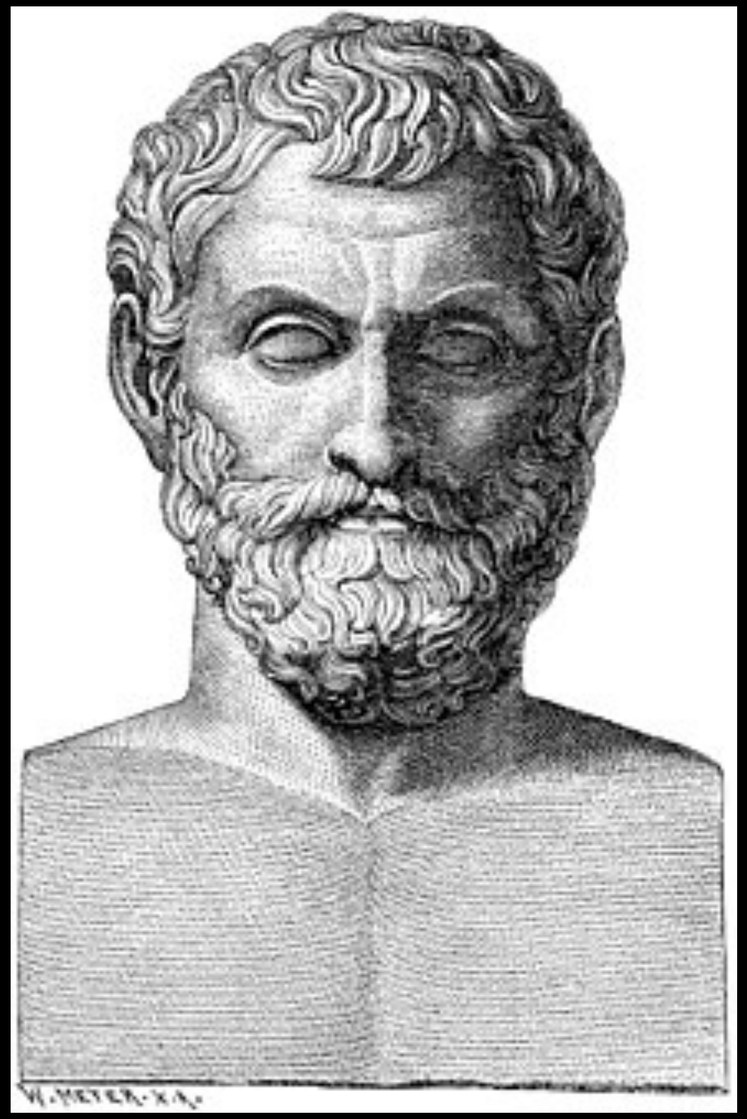
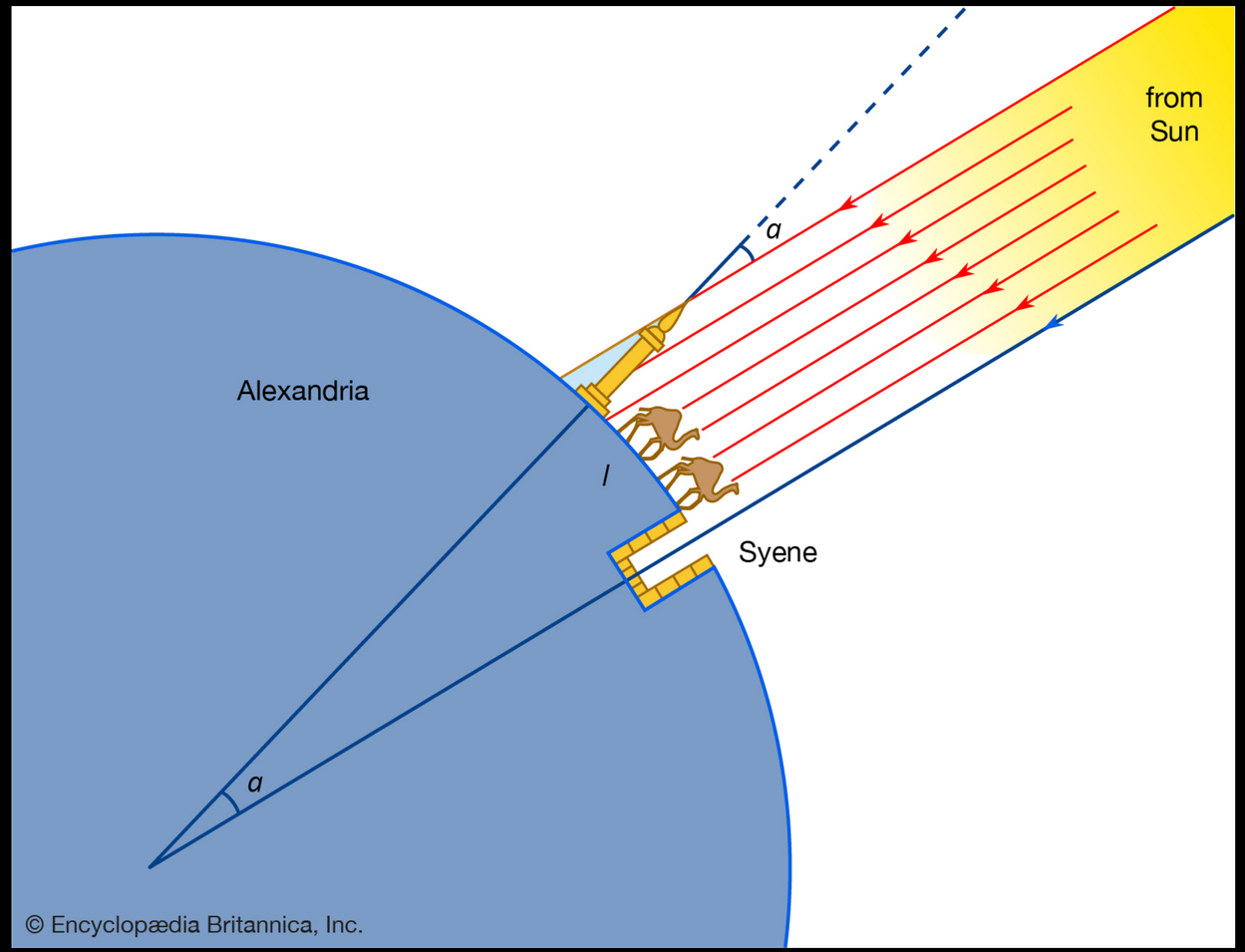
Thales

- Thales learned from his travels to Egypt and added to his own results.
- For instance, he proved that isoceles triangles had two equal angles.
- That to know a triangle it is enough to know one side and two angles.
- This is a very useful property for sailors, because it can give distance of ship to shore.
- Heights of pyramids (and other structures) could be found usinf shadows and similarity of triangles.



Triangles

- Creating right angled triangles is a cornerstone of the construction industry. People have used Thales' (angle in a semi-circle is a right angle) and Pythagoras' results since time immemorial.
- Eratosthenes used such knowledge to calculate the circumference of the earth in 240 BC.
- He used the information that on summer solstice the sun falls directly on top of the Tropic of Cancer, so he could measure from Alexandria the angle the sun made and compare it with another city where the sun fell directly overhead.
- The full circumference is covered by 360 degrees, and so on.
- He also calculated the tilt of the earth.


from
Sun
a
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Trigonometry

- Then came trigonometry, angles and ratios were useful for navigators who used sine and cosine tables to correct courses and carried tools such as quadrants.
- Making maps was an important skill and was aided by geometrical projections (for example the Mercator projection).
- Modern map making is far more complicated, sycg as GPS takes into account the movement of the satellite, its orbit, rotation of the earth, axis of its rotation and many other aspects.

Architecture & Art

- Calculating volumes of cylinders, domes, etc. was very useful in architecture.
- For that approximate values of Pi were required.
- Other advances in art resulted via advances in perspective and optics. (Cf. Brunelleschi's portrait of the Baptistry.)
- Renaissance art was revolutionized by the ability to make accurate lifeline paintings.
- Nowadays architects use CAD to do the mathematics.
- Visual effects designers for games and movies use geometry all the time.
- But all the skills used by people like Erathosthenes, Heron, Euclid, etc. are embedded in software, so we do not really see the application of geometry.


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HAROLD

Algebra

- Algebra was originally just word problems. It was concerned with practical issues.
- In the 16th century it moved to symbols (via François Viète).
- Nowadays every practical thing we do uses a lot of equations in the background.
- Most of high school algebra is actually geometry.
- For example the quadratic eqution $x^2 + b = A$ can be solved using geometry (completing the square).

60. PROBLEM FROM LILAVATI

Here is an ancient problem from Bhaskaracharya's Lilavati:

Beautiful maiden, with beaming eyes, tell me which is the number that, multiplied by 3, then increased by three-fourths of the product, divided by 7, diminished by one-third of the quotient, multiplied by itself, diminished by 52, the square root found, addition of 8, division by 10 gives the number 2?

Well, it sounds complicated, doesn't it? No, not if you know how to go about it.

Algebra

- Learning to think in abstract terms, to handle algebra is an important cognitive tool in the development of human cognition.
- Different degree two equations give rise to objects occuring naturally (such as circles, ellipses - orbits, parabolas - trajectory, etc.).
- So, algebra is very essential in studying various physical phenomenon. In fact, it was used extensively by Newton in his Principia (along with geometry).
- Knowledge of solutions of quartic, cubic equations are essential to make curvy surfaces (such as in cars, buildings and even in video games).

Abstract Algebra

- Work of Galois was intrumental in establishing abstract lagebra.
- This has continued to the modern day.
- Work by Emmy Noether is instrumental in physics. Her work has led to predictions of elementary particles and gave a understanding of our universe.
- Other applications: Google Search, Nash equilibrium, etc.

Other Developments

- Logarithm, used by astronomers Kepler to give his third law.
- Slide rules (which are an offshot of logarithms) had been in use even for the Apollo missions and Fermi’s experiments (well into the middle of the last century).
- Other developments such as Imaginary numbers, solutions of cubic equations, quantum mechanics, quaternions & Octonions have had so much use, for example in the music industry (via Fast Fourier transforms, etc.).

Even More Developments

- Calculus opened up a whole new world.
- It is now used in making wings of aeroplanes, infectious disease modelling, and almost everywhere physics is required.
- Information theory, the basis of modern day communications.
- Statistics, used in almost every branch of study nowadays.
- …. and much more.

Thank You!