MAT165: PROBLEMS ON PARITY

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(1) On a chessboard a knight starts at $a1$ and after several moves reaches $a1$ again. Show that the knight has moved an even number of times.

(2) The product of 22 integers is 1, show that their sum cannot be 0.

(3) (a) If the product of 2002 integers equals 1, can their sum be zero?
   (b) What if you replace 2002 by any positive even integer?
   (c) If you replace 2002 by any positive integer, what are the possible values their sum can take?

(4) (a) A man starts at 0 on the number line, and moves either left or right on each move. If he moves 1 unit on the first move, 2 units on the second move, 3 units on the third move, and so on, is it possible for him to return to the starting point after 2023 moves?
   (b) What if you were to replace 2023 by any fixed positive integer n?
   (c) For each positive integer n, what are the possible locations the man could be at?

(5) The numbers $1, 2, 3, \ldots, 2023$ are written on a blackboard. Two of the numbers are erased from the blackboard and replaced by their positive difference. This procedure is carried out until only one number remains. Can this remaining number possibly be 0?

(6) (a) Can you completely tile a 10×10 square board with 1×2 tiles if you remove squares at diagonally opposite corners of the square board?
   (b) Can a knight start at square $a1$ of a chessboard and end at the diagonally opposite square $h8$ covering each of the remaining squares exactly once on the way?
   (c) There are 2023 students are seated in rows and columns in a class. Can they be reseated so that each student occupies a seat which is either in a row or a column adjacent to their original seat?

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