MAT165: PROBLEMS ON COMBINATORICS

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(1) How many three-digit numbers can be formed using the digits 1 to 9, if repetition is not allowed?

(2) A password consists of 2 letters followed by 3 digits. How many such passwords are possible if letters and digits can be repeated?

(3) How many 4-letter words (not necessarily meaningful) can be formed using the letters A, B, C, D, E if no letter is repeated?

(4) A fruit basket contains 4 apples, 3 oranges, and 2 bananas. How many different ways can you select 3 fruits?

(5) In a quiz competition, there are 6 questions. Each question has 4 options and only one correct answer. How many different ways can you answer all the questions?

(6) A committee of 2 boys and 2 girls is to be formed from 5 boys and 4 girls. How many such committees are possible?

(7) How many 5-digit numbers can be formed using only even digits (0, 2, 4, 6, 8) such that the number does not start with 0?

(8) From 7 men and 5 women, how many different committees of 4 people can be formed such that at least one woman is on the committee?

(9) How many 4-digit numbers with distinct digits are there such that the digits are in strictly increasing order?

(10) A student must answer 5 out of 7 questions in an exam. In how many ways can the student choose the questions if:
   (a) There are no restrictions?
   (b) Question 1 must be included?

(11) How many ways are there to distribute 10 identical balls into 4 distinct boxes such that no box is empty?

(12) Justify the convention that $0! = 1$ and $n^0 = 1$ for all integers n.

(13) How many ways are there to distribute n identical balls in k identical boxes?

(14) How many ways are there to distribute n identical balls in k identical boxes if each box has to get at least one ball?

(15) How many ways are there to distribute n identical balls in k identical boxes if each box has to get at most one ball?

(16) How many ways are there to distribute n distinguishable balls in k identical boxes?

(17) How many ways are there to distribute n distinguishable balls in k identical boxes if each box has to get at least one ball?

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(18) How many ways are there to distribute n distinguishable balls in k identical boxes if each box has to get at most one ball?
(19) How many ways are there to distribute n identical balls in k distinguishable boxes?
(20) How many ways are there to distribute n identical balls in k distinguishable boxes if each box has to get at least one ball?
(21) How many ways are there to distribute n identical balls in k distinguishable boxes if each box has to get at most one ball?
(22) How many ways are there to distribute n distinguishable balls in k distinguishable boxes?
(23) How many ways are there to distribute n distinguishable balls in k distinguishable boxes if each box has to get at least one ball?
(24) How many ways are there to distribute n distinguishable balls in k distinguishable boxes if each box has to get at most one ball?
(25) (a) Find the number of ways to place n rooks on an $n \times n$ chess board so that no two of them attack each other.
(b) How many ways are there to place some rooks on an $n \times n$ chess board so that no two of them attack each other?
(26) Prove that in any company of people, two people know the same number of people in that company.
(27) Prove that, for $n > 0$, $2\binom{2n-1}{n} = \binom{2n}{n}$.
(28) Prove that, for all $k \leq n$ we have $\binom{k}{2} + \binom{n-k}{2} + k(n-k) = \binom{n}{2}$.
(29) Let n be a positive integer, prove the following identities:
(a) $\sum\limits_{k=1}^{n} \binom{n}{k}^2 = n\binom{2n-1}{n-1}$,
(b) $\sum\limits_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$.
(30) For all $n, m, k \in \mathbb{N}$, prove that

$$\binom{m+n}{k} = \sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i}.$$

(31) Consider $n \times n$ matrices where the entries are from the set $\{0, 1, -1\}$ such that all the row sums and column sums of the matrix are both 1. Also, in any row or column, successive non-zero entries change sign.
(a) List down all such matrices when $n = 1, 2, 3$.
(b) Show that in the first row or first column there can be only one non-zero entry.
(c) Show that, if such a matrix is vertically symmetric then n must be odd.
(d) What is the middle column of such a matrix that is vertically symmetric?
(32) Prove the following identity, for all $n > 0$:

$$\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n.$$