# MAT165: PROBLEMS ON GRAPHS

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(1) A $4 \times 4$ chessboard has its corner squares missing. Can a knight travel around this board, and pass along each square exactly once, and end up on the same square it started on?
(2) Prove that the number of people who lived on earth and who have shaken hands an odd number of times in their lives, is even.
(3) A person X coming home from a place Y, said that they saw a lake with 7 islands, to each of which led either $1, 3$ or $5$ bridges. Is it true that at least one of these bridges lead to the shore of the lake?
(4) In a country there are 15 towns, each of which is connected to at least 7 others. Prove that one can travel from any town to any other town possibly through some towns in between.
(5) Prove that a graph with $n$ vertices, with degree at least $(n-1)/2$ is connected.
(6) In a certain country, 100 roads lead out of the city, and one can travel along these roads from any city to any other. One road is closed for repair, prove that one can still get from any city to any other.
(7) Prove that there doesn't exist a graph on 5 vertices with degree sequence $4, 4, 4, 4, 2$.
(8) Does there exist a graph with $2n$ vertices with degree sequence $1, 1, 2, 2, \ldots, n, n$?
(9) Is is true that 2 graphs must be isomorphic, if
    (a) they both have $|V| = 10$ and $\deg(v) = 9$?
    (b) they both have $|V| = 8$ and $\deg(v) = 3$?
(10) Prove that in any tree with at least one edge, there exist a vertex which is the endpoint of only one edge.
(11) All vertices in a graph have degree 3, prove that it has a cycle.
(12) There are 7 lakes in a city, which are connected by 10 canals. How many islands are there?
(13) Prove that $2|E| \geq 3|F|$.

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