**Enrolment No. _____**

# AHMEDABAD UNIVERSITY

## SCHOOL OF ARTS & SCIENCES

## UNDERGRADUATE PROGRAMMES

2025-2026 MONSOON SEMESTER

MID SEMESTER EXAMINATION

# MAT165 GATEWAY TO ABSTRACT REASONING

Date: 02-12-2025 Total Marks: 20

Time: 02.00 pm – 04.00 pm No. of printed pages: 03

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General Instructions:

1. Students must carry their identity card during the examination.
2. No break is allowed during the examination under any circumstances. In case of an emergency, with prior permission of the invigilator and an escort, a break of maximum five minutes is allowed.
3. No unauthorized devices, such as mobile phones, any kind of watches, other gadgets, or any kind of material is allowed on person.
4. Any violation of examination rules and an intent of malpractice will lead to strict disciplinary action, including a possible expulsion from the university.

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**Questions**

1. **Each of the following questions is worth 0.5 marks. Attempt any two.**
   **In each, if true, give a reason (in 1 sentence); if false, give a counterexample.**

   (a) True or False: Every even square is divisible by 8.

   (b) True or False: If $a \mid (b + c)$, then $a \mid b$ or $a \mid c$.

   (c) True or False: The number of ways of writing 5 as a sum of positive integers is 7 (where order doesn't matter, that is, 4+1 is the same as 1+4).

2. **Each of the following questions is worth 1 mark. Attempt any two.**

   (a) The product of 22 integers is 1; show that their sum cannot be 0.

   (b) Can you completely tile a $10 \times 10$ square board with $1 \times 2$ tiles if you remove squares at diagonally opposite corners of the square board? Explain your reasoning.

   (c) A committee of 2 boys and 2 girls is to be formed from 5 boys and 4 girls. How many such committees are possible? How many committees are possible if there has to be at least 1 girl in every committee?

3. **Each of the following questions is worth 2 marks. Attempt any two.**

   (a) Show that $\frac{1}{a(a+b)} + \frac{1}{(a+b)(a+2b)} + \cdots + \frac{1}{(a+nb-b)(a+nb)} = \frac{n}{a(a+nb)}$, where $a$ and $b$ are natural numbers.

   (b) Find the remainder when $4444^{4444}$ is divided by 9.

   (c) Prove that in any company of people, two people know the same number of people in that company. (Here, we assume that any pair of people has only two possible relationships: either they know each other or they don't.)

4. **Each of the following questions is worth 3 marks. Attempt any two.**

   (a) Let $n$ be a positive integer, then prove that $\sum_{k=1}^{n} \binom{n}{k}^2 = n \binom{2n-1}{n-1}$. Avoid using mathematical induction for full credits.

   (Hint: The right-hand side counts the number of ways of picking $n$ people out of $2n$ people, such that one of the chosen people is special. Why? Try to explain this for the left-hand side as well.)

   (b) Prove that if both $a$ and $b$ are odd integers, then $16 \mid a^4 + b^4 - 2$.

   (c) A partition of $n$ is a sequence of non-increasing integers $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k$ such that all the $\lambda_i$'s (which are called parts of the partition) sum up to $n$. Show that the number of partitions of $n$ with the largest part $k$ equals the number of partitions of $n$ with exactly $k$ parts.

5. **Each of the following questions is worth 3.5 marks. Attempt any two.**

(a) There are 2023 students seated in rows and columns in a class. Can they be reseated so that each student occupies a seat which is either in a row or a column adjacent to their original seat? Explain your reasoning.

(b) A basket contains $n$ mangoes. If it is distributed among 2 people, 1 mango is left. If we distribute among 3, 5, or 7 people, we are left with 2, 4, or 6 mangoes, respectively. However, we can distribute equally among people. Find the smallest such $n$.

(c) Show that if $x, y, z$ are integers satisfying $x^2 + y^2 = z^2$, then $xyz \equiv 0 \pmod{60}$.