# MAT165 ASSIGNMENT 4

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**Last Date of Submission.** 23 January 2026 **before** the lecture starts.

**Instructions.**

- You can discuss the problems with any of your class-mates. In fact, I encourage you to talk to your friends and come up with the solutions together.
- Avoid using AI or web search to arrive at the solutions. This way would be easy, but you will learn very little.
- Write down the solutions in A4 sized sheets of paper (either blank or dotted), and staple them before submission. Use either black or blue ink for writing the solutions. **Failure to follow this will result in an immediate score of zero.**
- If you are unable to solve a problem, then mention what approach you took and how that did not work out. Sometime in mathematics, false starts can lead to promising avenues in other directions.

**Questions.**

(1) In a $3 \times 3$ chessboard there are two white knights in the first row at the two corners and there are two black knights in the third row at the two corners. Is it possible to make legal chess moves and transform this arrangement into an arrangement where the first row has a white knight in the left corner and a black knight in the right corner, while the third row contains a black knight in the left corner and a white knight in the right corner? If yes, explain the sequence of moves you make, and if not, then give a justification.

(2) In the lectures, we saw that in any collection of 6 people, where every pair of them are either mutually friends or mutually enemies, we can always find 3 people such that all of them are either friends with each other or enemies. Show that this result will no longer be true if we had only 5 people.

(3) 65 students gave the mid-semester examination for this course. Show that there must be a sequence of 9 students such that there marks are either increasing or decreasing.

(4) Every point in a plane is either red, green, or blue. Prove that there exists a rectangle in the plane such that all of its vertices are the same color.

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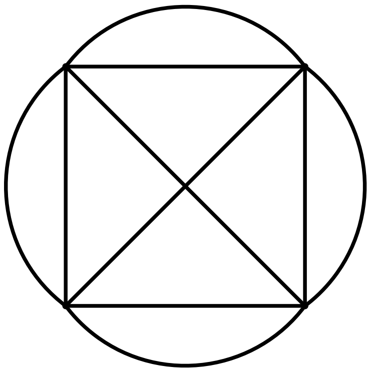
*Date*: 18 January 2026.

(5) Look at the picture below. Can you draw this without lifting the pen/pencil from the paper such that every line is traced exactly once and you come back to your starting point? If yes, give a sequence of steps to accomplish this, if not, give a justification.



(6) How many non-attacking kings can you place in an $n \times n$ chessboard?