MAT165 ASSIGNMENT 6

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Last Date of Submission. 13 February 2026 **directly to the TAs**.

Instructions.

- You can discuss the problems with any of your class-mates. In fact, I encourage you to talk to your friends and come up with the solutions together.
- Write down the solutions in A4 sized sheets of paper (either blank or dotted), and staple them before submission. Use either black or blue ink for writing the solutions. **Failure to follow this will result in an immediate score of zero.**

Questions.

(1) Let G be a simple graph with n vertices ($n \geq 2$). Prove that if the degree of every vertex is at least $\frac{n-1}{2}$, then the graph G is connected.

(2) A specific tree T contains vertices of only two types: degree 1 and degree 4. If T has exactly 10 vertices of degree 4, how many vertices of degree 1 (leaves) does it have?

(3) Is it possible to draw a connected graph with 6 vertices such that the degrees of the vertices are 2, 2, 3, 3, 4, 5 without lifting your pen or retracing an edge?

(4) A graph G is called "self-complementary" if it is isomorphic to its complement $\bar{G}$. (The complement $\bar{G}$ has the same vertices as G, but two vertices are connected in $\bar{G}$ if and only if they are *not* connected in G). Show that if G is self-complementary with n vertices, then $n \equiv 0$ or $1 \pmod 4$.

(5) Show that the sets $(0, 1)$, $(0, 1]$, $[0, 1)$, and $[0, 1]$ have the same cardinalities.

(6) Let $A_1, A_2, A_3, \ldots$ be a countable collection of countable sets. Prove that their union $S = \bigcup_{n=1}^{\infty} A_n$ is also countable.

(7) Let S be the set of all infinite sequences $(x_1, x_2, x_3, \ldots)$ where each $x_i \in \{0, 1\}$. Is S countable or uncountable? Prove your answer.

(8) Let F be the set of all **finite** subsets of the natural numbers $\mathbb{N}$. Is F countable or uncountable?

(9) Let I be the set of all **infinite** subsets of the natural numbers $\mathbb{N}$. Is I countable or uncountable?

(10) Let D be the set of all strictly decreasing sequences of natural numbers (e.g., $10, 5, 2, 1$). Is D countable?

Date: 05 February 2026.