



Ahmedabad University
School of Arts and Sciences
Elementary Number Theory & Cryptography
(MAT215) - Part A

Mid-Semester Examination, Date: 27 September 2025

Timing: up to 03:00 PM

Monsoon 2025

Maximum points: 10

Instructions

- You are allowed to bring only the following items with you: a calculator, stationery, and one A4-sized sheet of paper with anything written on it, double-sided (this doesn't mean two sheets, it means only one sheet, and it has to be A4 size; anything is not permissible).
- Once you submit Part A, you will get the Part B questions.
- You can do any combination of questions totaling 10 points. Anything above that will not be evaluated.
- Malpractice (including, but not limited to, bringing unauthorized things to the exam room, talking with others, prompting, and using electronic devices) will result in an immediate NP grade.

Best wishes

Questions

The quantities written in small letters, such as a, b, n , etc. will always be integers, unless otherwise mentioned.

Question 1 to 12 are worth 0.5 point each. Write only the answer for these questions, no justification is required unless asked for specifically.

1. What is $\gcd(-12, -30)$?
2. Prove or disprove: if $a|(b+c)$, then either $a|b$ or $a|c$.
3. If $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$, then what is $\gcd(a, bc)$?
4. What are the possible values of $\gcd(a+2b, 2a+b)$?
5. Which of the following Diophantine equations cannot be solved:
 - (a) $6x + 51y = 22$,
 - (b) $33x + 14y = 115$, and
 - (c) $14x + 35y = 93$.
6. Give an example to show that $a^2 \equiv b^2 \pmod{n}$ doesn't always imply $a \equiv b \pmod{n}$.

7. What is the remainder when 7 divides $2^{50} + 41^{65}$?
8. For all $n \geq 1$, if we have

$$\binom{2n}{n} = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots 2n} \times 2^x.$$

Then, what is the value of x ?

9. What is the value of $\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + \cdots + 2^n\binom{n}{n}$?
10. Is an integer of the form $3j + 2$ necessarily of the form $6k + 5$? Justify.
11. Find all prime numbers that divide $50!$.
12. How many zeroes does $2025!$ end in?

Questions 13 to 18 are worth 1 point each. Write 1-2 lines of justification wherever applicable, only writing the final answer will not fetch you full points.

13. What is the remainder when $2 \cdot (26!)$ is divided by 29?
14. Solve: $5x \equiv 2 \pmod{26}$, for x .
15. Prove that $1835^{1910} + 1986^{2061} \equiv 0 \pmod{7}$.
16. Show by any method that $15 | (2^{4n} - 1)$ for all $n \geq 1$.
17. Verify that $0, 1, 2, 2^2, 2^3, \dots, 2^9$ form a complete set of residues modulo 11.
18. Among all the results mentioned so far in the lectures, state your favorite. Give an example to illustrate the result you have stated.