Ahmedabad University

School of Arts and Sciences

Elementary Number Theory & Cryptography (MAT215)

Quiz 1, Date: 25 August 2025

Timing: 03:15 PM to 03:45 PM | Monsoon 2025 | Maximum points: 10

Instructions

- There are three questions, you can choose to do any number of them, in any order.
- Each question is worth 5 points.
- If you solve more than 2 questions, then your total score will still be capped at 10 points, even if the total for all three of your solutions is greater than 10.

Best wishes

Questions

1. Prove that for $n \in \mathbb{N}$,

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

2. Apply the Euclidean algorithm to find $\gcd(252, 198)$.
3. Show that the cube of an integer is of the form $7k$ or $7k \pm 1$, for some $k \in \mathbb{Z}$.

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Questions

1. Show that for all $n \geq 1$,
$$\prod_{k=1}^{n}\left(1+\frac{1}{k}\right)=n+1.$$
2. Compute $\gcd(45, 60)$ directly by listing divisors. Verify the result by checking linear combinations of 45 and 60.
3. If an integer is simultaneously a cube and a square, what form does it necessarily have to be of?

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1. Show that for all $n \geq 1$, $n! \geq 2^{n-1}$.
2. Show that $\gcd(a, b, c) = \gcd(\gcd(a, b), c)$.
3. Prove that if both a and b are odd integers, then $16 \mid a^4 + b^4 - 2$.

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Questions

1. Prove that, for all $n \geq 1$,

$$1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1.$$

2. Apply the Euclidean algorithm to find $\gcd(544, 119)$.

3. Prove or disprove: the sum of the squares of two odd numbers cannot be a perfect square.

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Questions

1. Prove that, if $1 + a > 0$, then $(1 + a)^n \geq 1 + na$, for all $n \geq 1$.
2. Apply the Euclidean algorithm to find $\gcd(1234, 4321)$ and express it as a linear combination of 1234 and 4321.
3. Prove or disprove: the product of four consecutive integers is 1 less than a perfect square.

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1. Prove that for all $n \in \mathbb{N}$,
$$7 \mid (8^n - 1).$$
2. Find $\gcd(101, 462)$ using the Euclidean algorithm.
3. If $\gcd(a, b) = 1$, then prove that $\gcd(a^2, b^2) = 1$.

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Questions

1. Show that for all $n \in \mathbb{N}$,
$$11 \mid (10^{2n+1} + 1).$$
2. Compute $\gcd(414, 662)$ and write it as a linear combination of the two numbers.
3. Verify that $2^{35} - 1$ is divisible by 31 and 127.

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Questions

1. Prove that for all $n \geq 1$, $6 \mid (n^3 - n)$.
2. Determine $\gcd(1234, 4321)$ by the Euclidean algorithm.
3. If $a \mid bc$, then show that $a \mid \gcd(a, b)\gcd(a, c)$.

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Questions

1. Show that for all $n \geq 1$, the integer $5^{2n} - 1$ is divisible by 24.
2. Compute $\gcd(91, 287)$ and show the steps.
3. What is the maximum number you can guarantee that will always divide the product of four consecutive integers? Prove your assertion.

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Best wishes

Questions

1. Show that for all $n \geq 1$,

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$

2. Use the Euclidean algorithm to determine whether $221x + 91y = 7$ has integer solutions. If so, find them.
3. If a is an integer not divisible by 2 or 3, then show that $24 \mid (a^2 + 23)$.

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Questions

1. Prove that for $n \geq 1$,

$$\sum_{k=1}^{n} k \cdot 2^k = (n-1)2^{n+1} + 2.$$

2. Use the Euclidean algorithm to compute $\gcd(899, 493)$.

3. For any integer a, what is $\gcd(5a+2, 7a+3)$? Prove your assertion.

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Questions

1. Prove that for all $n \geq 2$,
$$\left(1 + \frac{1}{n}\right)^n < 3.$$
2. Use the Euclidean algorithm to compute $\gcd(1001, 143)$.
3. State the division algorithm, and mention the major steps used in its proof, as done in the lecture. (You do not need to write down the proof, just the basic ideas.)

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Questions

1. Prove that for all $n \geq 0$,
$$9 \mid (10^{2n} - 1).$$
2. Determine $\gcd(867, 255)$.
3. State the well-ordering principle, and mention one of its consequences that we discussed in the lectures.

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Questions

1. Show that for $n \geq 1$,
$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}.$$

2. Show that if a and b are positive integers, then the Euclidean algorithm produces the same sequence of remainders when applied to (a, b) and (b, a).

3. Given two integers a, b, if there exists integers x, y such that $ax + by = \gcd(a, b)$, show that $\gcd(x, y) = 1$.

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Questions

1. Let $\{a_n\}$ be defined by $a_1 = 1$, $a_{n+1} = 2a_n + 1$. Prove that

$$a_n = 2^n - 1 \quad \text{for all } n \geq 1.$$

2. Apply the Euclidean algorithm to determine $\gcd(867, 255)$. Then use it to solve

$$867x + 255y = 51$$

in integers.

3. Prove that for any integers a, n, we have $\gcd(a, a + n) \mid n$.

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Best wishes

Questions

1. Show that for all $n \geq 1$,
$$\binom{2n}{n} \leq 4^n.$$
2. Find $\gcd(414, 662)$ using the Euclidean algorithm. Then solve the equation
$$414x + 662y = \gcd(414, 662)$$
in integers.
3. For any integer a, what is the maximum number that is guaranteed to divide one of the integers $a, a+2, a+4$. Prove your assertion.