

MAT631 ASSIGNMENT 1

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Last Date of Submission. 22 January, 2025 before 1430.

Instructions. Please read the instructions on the course website carefully before submitting your solution(s).

Questions.

- (1) (**Shubham**) Is the lexicographic ordering (that we discussed in Lecture 2) a partial ordering? Explain your reasoning.
- (2) (**Anant**) Show that the q -binomial coefficient $\binom{n}{k}_q$ is a polynomial in q .
- (3) (**Saikat**) Complete the proof of the the fact that

$$\sum_{\sigma \in S_n} q^{\text{inv}(\sigma)} = [n]_q!.$$

- (4) (**Anubhav**) Let $E(n, k)$ be the number of permutations in S_n with $k - 1$ descents. Then prove that for all $k, n \in \mathbb{N}$ with $k \leq n$, we have

$$E(n, k + 1) = (k + 1)E(n - 1, k + 1) + (n - k)E(n - 1, k).$$

- (5) (**Kanak**) Let $E(n, k)$ be as defined in the previous question. We set $E(0, 0) = 1$ and $E(n, 0) = 0$ for $n > 0$. Then, show that for all $n \geq 0$ and any $z \in \mathbb{C}$, we have

$$z^n = \sum_{k=1}^n E(n, k) \binom{z + n - k}{n}.$$