MAT631 PROBLEM SET 3

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Instructions. Please read the instructions on the course website carefully before submitting your solution(s).

Questions.

(1) **(Shubham)** Show that the lexicographic order on a partition is a linear extension of the dominance order, that is, $\lambda \preceq \mu$ implies that $\lambda \leq \mu$.

(2) **(Anant)** Show that the dominance order is "symmetric" with respect to conjugation. That is, show that $\lambda \prec \mu$ iff $\mu' \prec \lambda'$.

(3) **(Saikat)** Show that the descent set of the word u us the same as the descent set of the word $\mathrm{mn}(u)$.

(4) **(Anubhav)** For $\sigma \in S_n$, show that

$$\mathrm{mn}(\sigma) = 0^n + \sum_{i \in \mathrm{Des}(\sigma^{-1})} (e_{\sigma^{-1}(i+1)} + \cdots + e_{\sigma^{-1}(n)}).$$

(5) **(Kanak)** Explain why the algorithm to find the minimization of a word (as discussed in the lecture) works.