MAT631 PROBLEM SET 4

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Last Date of Submission. 10 February, 2025.

Instructions. Please read the instructions on the course website carefully before submitting your solution(s).

Questions.

(1) **(Shubham)** Define the coinversions of a word $w = w_1 w_2 \cdots w_n$, denoted by $\mathrm{coinv}(w)$ to be the number of pairs (i, j) with $1 \leq i < j \leq n$ and $w_i < w_j$. Prove that

$$\sum_{w \in R(0^a 1^b)} q^{\mathrm{coinv}(w)} = \binom{a+b}{a}_q.$$

(2) **(Anant)** Given a word $w = w_1 w_2 \cdots w_n$, let $\mathrm{comaj}(w)$ be the sum of all $i < n$ with $w_i < w_{i+1}$, and let $\mathrm{rlmaj}(w)$ be the sum of $n - i$ for all $i < n$ with $w_i > w_{i+1}$. Calculate $\sum_{w \in S_n} q^{\mathrm{comaj}(w)}$ and $\sum_{w \in S_n} q^{\mathrm{rlmaj}(w)}$.

(3) **(Saikat)** Prove the q-analogue of the negative binomial theorem: for all $n \in \mathbb{N}$, we have

$$\frac{1}{(a;q)_n} = \sum_{k=0}^{\infty} \binom{k+n-1}{k}_q a^k.$$

(4) **(Anubhav)** Solve exercise 8-66 (a), 8-66 (c), and 8-66 (d) of the reference book by Loehr.

(5) **(Kanak)** Solve exercise 8-66 (b) and 8-67 (a) of the reference book by Loehr.

Date: 02 February 2025.