

MAT631 PROBLEM SET 5

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Instructions. Please read the instructions on the course website carefully before submitting your solution(s).

Questions.

- (1) (**Shubham**) Complete the proof of the hook-length formula (as mentioned in the lecture).
- (2) (**Anant**) Generalize the construction in Viennot's shadows to billetter words and SSTs.
- (3) (**Saikat**) Prove that there is a bijection between pairs (τ, v) with τ a SST of shape λ and v is a non-decreasing word, and pairs (τ', λ) where τ' is a tableau of shape μ such that $\mu \setminus \lambda$ is a horizontal strip.
- (4) (**Anubhav**) Give a detailed proof of the following formula:

$$h_n = \sum_{\mu \vdash n} \frac{p_\mu}{z_\mu}.$$

- (5) (**Kanak**) Give a detailed proof of the following formula:

$$e_n = \sum_{\mu \vdash n} \frac{(-1)^{n-\ell(\mu)} p_\mu}{z_\mu}.$$