AHMEDABAD UNIVERSITY
SCHOOL OF ARTS & SCIENCES
GRADUATE PROGRAMMES
2025-2026 MONSOON SEMESTER
END SEMESTER EXAMINATION
MAT730 COMBINATORIAL REPRESENTATION THEORY

Date: 01-12-2025   Total Marks: 30
Time: 02.00 pm – 04.00 pm   No. of printed pages: 02

General Instructions:

1. Students must carry their identity card during the examination.
2. No break is allowed during the examination under any circumstances. In case of an emergency, with prior permission of the invigilator and an escort, a break of maximum five minutes is allowed.
3. No unauthorized devices, such as mobile phones, any kind of watches, other gadgets, or any kind of material is allowed on person.
4. Any violation of examination rules and an intent of malpractice will lead to strict disciplinary action, including a possible expulsion from the university.

Course Specific Instructions:

1. Any result discussed in the lectures or assignments can be assumed without proof.
2. All questions are worth **6 points each**. You can choose to **do any five (05) questions**.

Questions

1. (a) Let G be a finite group and χ and ξ be characters; then show that
$$\left\langle \chi, \xi \right\rangle = \frac{1}{|G|} \sum_{g \in G} \chi(g)\xi(g^{-1}).$$

   (b) Give a complete description of the character table for S_3.

2. If $\lambda \vdash n$, then show that M^λ it is cyclic and generated by any given λ-tabloid. What is the dimension of M^λ?

3. Show that the R-S map is a bijection. That is, show that the inverse map is well-defined.

4. Let π^r denote the reverse of the permutation π. Then show that, if $P(\pi) = P$, we have $P(\pi^r) = P^t$.

5. Show that the Schur functions are symmetric using the tableaux definition of Schur functions.

6. Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_\ell)$, then show that $s_{\lambda'} = det(e_{\lambda_i - i + j})_{1 \leq i, j \leq \ell}$.