Indian Institute of Information Technology Senapati, Manipur

Assessment-II, January 2023

Course Title: **Mathematics-1**	Course Code: **MA1011**
Semester: I	Maximum Marks: 25
Date of Examination: 16.01.2023	Time: 1 hour

Part A $(5\times 2=10\,\text{marks})$

1. Write the properties of Eigen values and Eigen vectors (any two each).

2. Using Cayley-Hamilton theorem, Compute A^3, where $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$.

3. Using properties of determinant, prove that $\begin{vmatrix} x & y & z\\ x^2 & y^2 & z^2\\ x^3 & y^3 & z^3\end{vmatrix}=xyz(x-y)(y-x)(z-x)$.

4. If $\sum_{k=1}^{\infty}a_k=A$ and $\sum_{k=1}^{\infty}b_k=B$. Is it true that $\sum_{k=1}^{\infty}a_kb_k=AB$? Justify your answer.

5. If $(a_n)\to 0$, then find the value of $\lim\left(\frac{(a_n+1)^2-1}{a_n}\right)$.

Part B $(3\times 5=15\,\text{marks})$

6. Using Cayley-Hamilton theorem, find the inverse of $A=\begin{pmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{pmatrix}$.

7. (a) Find a complete set of eigenvalues and eigenvectors for the matrix $A=\begin{pmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{pmatrix}$.

Does A have three orthogonal eigenvectors? Justify your answer. Finally, write down the vector $(2\ 0\ 1)^T$ as a combination of the eigenvectors of A.

OR

(b) Test the convergence of (i) $\frac{1}{4.7.10}+\frac{4}{7.10.13}+\frac{9}{10.13.16}+........$

(ii) $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\log\frac{n+1}{n}\right)$.

8. (a) If V and W be vector spaces and $T:V\to W$ be a linear transformation, prove that $nullity(T)+rank(T)=dim(V)$, here V is finite-dimensional.

OR

(b) Prove the followings:

(i) Similar matrices have the same eigen values.

(ii) If A and B be two similar matrices through the non-singular matrix M. If X is the eigen vector of A corresponding to the eigen values λ, then $M^{-1}X$ will be the eigen vector of B corresponding to λ.