1. Define Basis and Dimension of a vector spaces.

2. Show that the sequence defined recursively as $a_1 = 0,\ a_2 = \frac{1}{2},\ a_{n+1} = \frac{1}{3}\left(1 + a_n + a_{n-1}^3\right)$ for $n > 1$ converges and find its limit.

3. Find $\lim_{x \to 0} In\left(1 + x\right) / x$.

4. Prove that if f is continuous on a closed interval $[a,b]$ differentiable on the open interval (a, b) and if $f(a) = f(b) = 0$ then for a real number c, there is an x in (a,b) such that $cf(x) + f'(x) = 0$.

5. Give an example of a function which is not integrable and prove that it is not integrable.

6. Determine all values of k for which the following matrices are linearly independent

$$\begin{bmatrix} 1 & 0 \\ 1 & \text{k} \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ \text{k} & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}.$$

7. Test if $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ are similar matrices.

8. Find the values of x, if any, at which f is not continuous

a. $f(x) = 5x^4 - 3x + 7$ ii. $f(x) = \dfrac{x}{2x^2 + x}$

9. Find the Maclaurin series for $\dfrac{1}{1-x}$.

10. If $f(x) = x^{2/3}$, $a = -1$ and $b = 8$.

a. Show that there is no point c in (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

b. Explain why the result in part (i) does not contradict the Mean-Value Theorem.