MA1011: Problem Sheet 1 (Vector Spaces)

Dr Manjil P. Saikia

December 2022

Date of Submission

15 December 2022 by 1800 IST. If I am not in the office then please slide your submission under the door.

General Rules

- This problem sheet will not be graded BUT you must submit this to me so that we can understand the format of submission and feedback.

- You can work in groups and you are free to consult any material that you wish to, but please mention them when you write down your answers/solutions. You must also mention your roll number and branch at the top of your submission.

- Write legibly and in full sentences. Mathematics, like any other subject has its own style and diction. Our aim is to also learn how to write mathematics (and other technical materials).

Problems

1. Are the following subsets of $\mathbb{R}^3$ subspaces?

    (i) The plane of vectors (x, y, z) with $x = 1$?
    (ii) The plane of vectors (x, y, z) with $x = 0$?
    (iii) The plane of vectors (x, y, z) such that $x - y + 2z = 0$?

2. What is the smallest subspace of 3×3 matrices that contains all symmetric matrices and all lower triangular matrices? What is the largest subspace that is contained in both of those subspaces?

3. Let x and y be positive real numbers. Show that the set of all positive real numbers, with addition redefined to equal the usual xy and scalar multiplication redefined to equal to x^c, where c is a scalar, is a vector space. What is the "zero vector" for this vector space?

4. Let $\mathcal{M}$ be the space of all 2×2 matrices. Describe a subspace of $\mathcal{M}$ that contains the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, but not the matrix $\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$.

5. Describe the five types of subspaces of $\mathbb{R}^4$.

6. Let $\mathcal{C}$ be the unit disc in $\mathbb{R}^2$:
$$\mathcal{C} := \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}.$$
Is $\mathcal{C}$ a vector space? Justify your answer.

7. Let $\mathbb{P}_n[t]$ be the set of all polynomials in the variable t of degree at most n. Is $\mathbb{P}_n[t]$ a vector space? Justify your answer.

8. Let $\mathcal{M}_n$ be the set of all $n \times n$ matrices. Let W be the set of all matrices in $\mathcal{M}_n$ with trace equal to 0. Show that W is a subspace of $\mathcal{M}_n$.

9. Is $\mathbb{R}^2$ a subspace of $\mathbb{R}^3$? Justify your answer.

10. Give an example of a vector space which we have not discussed so far (either in the lectures or in this problem sheet).