

MA1011: Problem Sheet 10 (Differentiability)

Dr Manjil P. Saikia

January 2023

This problem sheet will NOT be graded, so you do not have to submit this to me. We will discuss the problems in the tutorials.

1. Find the derivatives of $\sinh x$ and $\cosh x$ using the chain rule.
2. Let $f : (-a, a) \rightarrow (-\pi/2, \pi/2)$ be defined by $f(x) = \arcsin(x/a)$, where $a > 0$. Find the derivative of f .
3. Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be two differentiable functions and assume that $f > 0$. Define $h : \mathbb{R} \rightarrow \mathbb{R}$ as

$$h(x) = f(x)^{g(x)}.$$

Prove that h is differentiable and then find its derivative.

4. Let $f : (a, b) \rightarrow \mathbb{R}$ be a differentiable function. Then, prove that
 - (a) $f(x)$ is increasing on (a, b) iff $f'(x) \geq 0$ for all $x \in (a, b)$, and
 - (b) if $f'(x) > 0$ for all $x \in (a, b)$, then $f(x)$ is strictly increasing.
5. Consider the function $f : (-\infty, 1) \rightarrow \mathbb{R}$ defined by $f(x) = \frac{1}{1-x}$. Determine the Taylor expansion of f about $x_0 = 0$.
6. Let $f : \mathbb{R} \rightarrow (-\pi/2, \pi/2)$ be defined by $f(x) = \arctan(x)$. Compute the Taylor expansion of the function of order 3 about $x_0 = 1$.
7. Prove the following infinite series expansion for $\alpha \neq 0$,

$$e^{\alpha x} = \sum_{k=0}^{\infty} \frac{1}{k!} (\alpha x)^k.$$

8. Estimate the value of e to 5 decimal places.