

MA1011: Problem Sheet 11 (Integration)

Dr Manjil P. Saikia

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This problem sheet will NOT be graded, so you do not have to submit this to me. We will discuss the problems in the tutorials.

1. Evaluate the following integrals:

(a) $\int_1^3 \frac{1}{x}(1 + \ln x)^5 dx,$

(b) $\int_a^t \cos^5 x dx,$

(c) $\int_a^t \sin^4 x \cos^4 x dx,$

(d) $\int_a^t \sin^2 x (\cos x)^{-3} dx,$

(e) $\int_a^t \frac{x^2 + 3}{2x^2 - 3x - 2} dx,$

(f) $\int_a^t \frac{e^x + 3}{e^x + 2 + 10e^{-x}} dx,$

(g) $\int_0^{\pi/2} \frac{1}{1 + \sin x} dx,$

(h) $\int_a^t \frac{x^3}{\sqrt{1-x^2}} dx,$

(i) $\int_a^t \sqrt{x^2 - 16} dx,$ and

(j) $\int_a^t \sqrt{x^2 + 3x + 3} dx.$

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} xe^{3x^2}, & x < -1, \\ \sec^2 x, & -1 \leq x < 1, \\ \cos^3 x, & x \geq 1. \end{cases}$$

Evaluate $\int_{-2}^3 f(x) dx.$

3. Evaluate $\int_0^1 e^{-x^2} dx$ accurate to four decimal places.

4. For which values of $\alpha \in \mathbb{R}$ does the integral $\int_0^1 x^\alpha dx$ converge?