

MA1011: Problem Sheet 2 (Matrices)

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Date of Submission

17 December 2022 by 1800 IST. If I am not in the office (F-7) then please slide your submission under the door.

General Rules

- This problem sheet will be graded, the numbers in the brackets denote the points for each question.
- You can work in groups and you are free to consult any material that you wish to, but please mention them when you write down your answers/solutions. You must also mention your roll number and branch at the top of your submission.
- Write legibly and in full sentences. Mathematics, like any other subject has its own style and diction. Our aim is also to learn how to write mathematics (and other technical materials).

Problems

1. How many mathematical operations are required in the usual matrix multiplication of two $n \times n$ matrices? Explain your reasoning. [**2 points**]
2. (a) Under what conditions is the square A^2 of a matrix defined? (b) Show that A and A^2 commute. (c) How many matrix multiplications are needed to compute A^n ? [**3 points**]
3. A square matrix N is called *nilpotent* if $N^k = 0$ for some $k \in \mathbb{N}$. Find a nilpotent matrix which is neither lower nor upper triangular. [**2 points**]
4. A *permutation matrix* is a matrix where each row and each column has only one non-zero entry and that non-zero entry equals 1. How many such matrices of order n exist? Justify your answer. [**1 point**]
5. Explain why a matrix with a row of all zeros does not have an inverse. [**2 points**]
6. Let A be an $n \times n$ matrix. Which is faster to compute, A^2 or A^{-1} ? Justify your answer. [**3 points**]