

MA1011: Problem Sheet 7 (Eigenvalues & Eigenvectors)

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Date of Submission

12 January 2023 by 1200 IST. If I am not in the office (F-7) then please slide your submission under the door. Make sure to staple any loose sheets of paper.

General Rules

- This problem sheet will be graded, the numbers in the brackets denote the points for each question.
- You can work in groups and you are free to consult any material that you wish to, but please mention them when you write down your answers/solutions. You must also mention your roll number, section and branch at the top of your submission.

Problems

1. Prove that a matrix is singular if and only if it has a zero eigenvalue. [**3 points**]
2. Find the eigenvalues and the corresponding eigenvectors of the matrix $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -2 \\ 2 & 2 & -1 \end{pmatrix}$. [**2+3 points**]
3. Can you conclude something about the complex eigenvalues and the corresponding eigenvectors of a matrix with real entries from the previous question? Explain your reasoning. [**3 points**]
4. Prove that the sum of the eigenvalues of a square matrix A equals its trace, and the product of the eigenvalues equals its determinant. [**2+2 points**]
5. Prove that the only eigenvalue of a nilpotent matrix is 0. [**4 points**]
6. State whether the following are true or false, with reason if true and counterexample if false:
 - (a) Every invertible matrix can be diagonalized.
 - (b) Every diagonalizable matrix can be inverted.
 - (c) Exchanging the rows of a 2×2 matrix reverses the signs of its eigenvalues.

[**2+2+2 points**]