Indian Institute of Information Technology (IIIT) Manipur
Assessment II, May 2023

Course Title: **Mathematics II** Course Code: **MA1012**

Semester: II (Sections A & B) Maximum Marks: 25

Date of Examination: 9 May 2023 Time: 60 minutes

Write legibly and show your full work to get credit.

Part A ($5 \times 2 = 10$ **marks**)

Instructions

- All questions are compulsory.

Questions

1. Given S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = \sqrt{x^2 + y^2}$. Parametrize S by using the spherical coordinates. **(2 marks)**

2. Using the second fundamental theorem of calculus, show that
$$\int_C 3x^2 dx + 2yzdy + y^2 dz = 2,$$
where C is a circular arc connecting the points $(0, 0, 0)$ and $(1, 1, 1)$. **(2 marks)**

3. Find the volume of the solid S bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$, $y = 2$ and the coordinate planes. **(2 marks)**

4. Define the curl of a vector field in terms of some cross product. Explain your notation completely for full credit. **(2 marks)**

5. Consider the surface $S : x^2 + y^2 + z^2 = 8$, $-1 \leq z \leq 2$. Find the unit outward normal to S. **(2 marks)**

Part A ($3 \times 5 = 15$ **marks**)

Instructions

- Question 6 is compulsory in this part. For questions 7 and 8, you can choose to do either part (a) or part (b).
- If you do both parts for a question then marks will be awarded only for the first answered part (which is not crossed-out), even if the solution is not complete.

Questions

6. Evaluate
$$\iiint_D \frac{z}{(x^2 + y^2 + z^2)^{3/2}} dV,$$
where $D = \{(x, y, z) : x^2 + y^2 + z^2 \leq 4a^2, z \geq a\}$. **(5 marks)**

7. (a) Evaluate the following using double integrals

$$\int_0^1 (\tan^{-1} \pi x - \tan^{-1} x) dx.$$

(5 marks)

OR

(b) Find the volume of the solid in the first octant bounded below by the surface $z = \sqrt{x^2 + y^2}$ and above by $x^2 + y^2 + z^2 = 8$ as well as the planes $y = 0$ and $y = x$. **(5 marks)**

8. (a) Evaluate the area of the region enclosed by the simple closed curve $x^{2/3} + y^{2/3} = 1$. **(5 marks)**

OR

(b) Let C be the parametric curve $R(t) = (\cos t, \sin t, \cos t + 4)$, $0 \leq t \leq 2\pi$ and

$$F(x, y, z) = (z^2 + e^z, 4x, e^z \cos^2 z).$$

Evaluate $\oint_C F \cdot dR$. **(5 marks)**