Indian Institute of Information Technology (IIIT) Manipur
Assessment I, April 2023

Course Title: **Mathematics II** Course Code: **MA1012**

Semester: II (Sections A & B) Maximum Marks: 25

Date of Examination: April 2023 Time: 60 minutes

Write legibly and show your full work to get credit.

Part A ($5 \times 2 = 10$ **marks**)

Instructions

- All questions are compulsory.

Questions

1. Given $f(x, y) = z, x = r\cos\theta$ and $y = r\sin\theta$. If $z = x^2 + 2xy$, then evaluate $\dfrac{\partial z}{\partial \theta}$ in terms of x and y. **(2 marks)**
2. Given $f : \mathbb{R}^3 \to \mathbb{R}$ and $X_0 \in \mathbb{R}^3$, show that $f_x(X_0)$ can be written as a directional derivative in the direction of some vector in $\mathbb{R}^3$. **(2 marks)**
3. A plane curve has the equation $y = f(x)$, where the function f is twice differentiable. What is the curvature of the curve at the point $(x, f(x))$? **(2 marks)**
4. Given $f : \mathbb{R}^3 \to \mathbb{R}$ be differentiable and $R(t) = (x(t), y(t), z(t)), t \in \mathbb{R}$ be a differentiable curve such that $f(R(t))$ attains its minimum at some t_0. Show that $\nabla f(R(t_0))$ is perpendicular to $R'(t_0)$. **(2 marks)**
5. State the mixed derivative theorem (also called Euler's theorem in some textbooks). **(2 marks)**

Part A ($3 \times 5 = 15$ **marks**)

Instructions

- Question 6 is compulsory in this part. For questions 7 and 8, you can choose to do either part (a) or part (b).
- If you do both parts for a question then marks will be awarded only for the first answered part (which is not crossed-out), even if the solution is not complete.

Questions

6. Find the local maxima, local minima and saddle point(s) for the following function **(5 marks)**

$$x^4 + y^4 - 2x^2 - 2y^2 + 4xy.$$

7. (a) Given $f : \mathbb{R}^n \to \mathbb{R}$ is defined by $f(x_1, x_2, \ldots, x_n) = x_1^2 x_2^2 \cdots x_n^2$ and $n \geq 2$. Using the Lagrange multiplier method, find the maximum value of f subject to the constraint $g(x_1, x_2, \ldots, x_n) = x_1^2 + x_2^2 + \cdots + x_n^2 = 1$. What is the minimum value of f? **(4+1 marks)**

OR

(b) Given $S \subset \mathbb{R}^3$ be an open convex set and $f : S \to \mathbb{R}$ be a differentiable convex function on S. Prove that for all $X, Y \in S$, we have **(5 marks)**

$$f(X) - f(Y) \geq (X - Y) \cdot \nabla f(Y).$$

8. (a) Given the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\frac{1}{\sqrt{x^2+y^2}}\right), & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Show that f_x and f_y are not continuous at $(0,0)$. Is f differentiable at $(0,0)$? **(2+2+1 marks)**

OR

(b) Consider the cone $z = x^2 + y^2$.

(I) Find the equation of the tangent plane to the cone at $(1, 1, \sqrt{2})$. **(3 marks)**

(II) Find an equation for the normal line to the cone at the point. **(2 marks)**