

Indian Institute of Information Technology Senapati, Manipur
Assessment-I, April 2023

Course Title: **Mathematics II**
Semester: II
Date of Examination: 12.04.2023

Course Code: **MA1012**
Maximum Marks: 25
Time: 1 hour

Part -A (2×5 = 10 marks)

1. Evaluate the limit: (i) $\lim_{\substack{x \rightarrow 1 \\ y \rightarrow 2}} \frac{2x^2 + y^2}{2xy}$ (ii) $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy}{y - x^2}; x \neq 0, y \neq 0$

2. Is the function $f(x, y) = \begin{cases} \frac{x}{\sqrt{x^2 + y^2}}, & \text{for } x \neq 0, y \neq 0 \\ 2, & \text{for } x = 0, y = 0 \end{cases}$ continuous at the origin?

Justify your answer.

3. Show that the function $f(x, y) = \begin{cases} 2x^2 + y, & (x, y) \neq (1, 2) \\ 0, & (x, y) = (1, 2) \end{cases}$ is discontinuous at (1, 2).

4. If $u = e^{xyz}$, find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$.

5. Write the statement of Euler's theorem on homogenous function.

Part -B (5×3 = 15 marks)

6. If $u = \sin^{-1} \sqrt{\frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} + x^{\frac{1}{2}}}}$ then show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12} \left\{ \frac{13}{12} + \frac{\tan^2 u}{12} \right\}$

7. (a) Using the method of Lagrange's multiplier, find the largest product of the numbers x, y and z when $x^2 + y^2 + z^2 = 9$

OR

(b) (i) Find the Jacobian's of Spherical polar coordinate.

(ii) Calculate $\frac{\partial(x, y)}{\partial(u, v)}$ if $u = x^2 - y^2$ and $v = 2xy$

8. (a) Expand $f(x, y) = 21 + x - 20y + 4x^2 + xy + 6y^2$ in Taylor's Series of maximum order about the point $(-1, 2)$.

OR

(b) Evaluate the following over the region R that is bounded by the graphs of the given equations

(i) $\iint_R x^3 y^2 dA; x = y, y = 0, x = 1$ (ii) $\iint_R (x+1) dA; x = y, x + y = 4, x = 0$.
