Review of MA1011 (abridged)

Functions from $\mathbb{R}$ to $\mathbb{R}^3$ ... called vector valued fns.
Functions from $\mathbb{R}^3$ or $\mathbb{R}^n$ to $\mathbb{R}$ .... fns of several variables.

Norm of a vector: $v = (x, y, z)$, $\|v\| = \sqrt{x^2 + y^2 + z^2}$.

$\|v - u\|$ is the distance betn v and u.

Scalar product: $v = (v_1, v_2, v_3)$, $u = (u_1, u_2, u_3)$

$$v \cdot u = v_1 u_1 + v_2 u_2 + v_3 u_3.$$

Projection of a vector: The projection of a vector A along the non-zero vector B is $\frac{A \cdot B}{B \cdot B} B$.

Angle: If θ is the angle betn vectors v and u, then

$$v \cdot u = \|u\| \, \|v\| \cos\theta.$$

Eqns of straight lines:

- The parametric repn of the st. line passing through p and parallel to a vector is given by $v - p = tu$, $t \in \mathbb{R}$.
- For $v = (v_1, v_2, v_3)$, $p = (p_1, p_2, p_3)$, $u = (u_1, u_2, u_3)$, the eqn is then $v_i = p_i + t u_i$ for $i = 1, 2, 3$.
- If $u_1, u_2, u_3 \neq 0$ then we have,

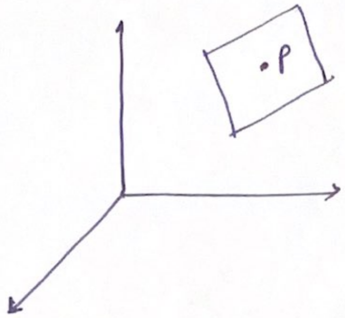
$$\frac{v_1 - p_1}{u_1} = \frac{v_2 - p_2}{u_2} = \frac{v_3 - p_3}{u_3}.$$

- If $u_1 = 0$ then the line is $v_1 = p_1$ and $\frac{v_2 - p_2}{u_2} = \frac{v_3 - p_3}{u_3}$.

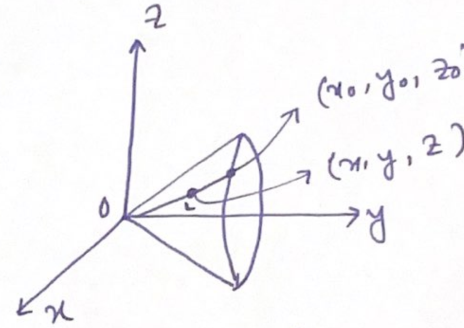
Eqn of a plane:

The set of pts $\{x : (x - p) \cdot v\} = \{x : x \cdot v = p \cdot v\}$ in $\mathbb{R}^3$ is the plane $\perp^r$ to the vector v and passing throug p.

eg:



• If $v = (v_1, v_2, v_3)$ and $p = (x_1, y_1, z_1)$ then the eq^n of the plane is, $(x, y, z) \cdot (v_1, v_2, v_3) = (x_1, y_1, z_1) \cdot (v_1, v_2, v_3)$.



Q. Find the eq^n of the right circular cone having vertex at $(0,0,0)$ and passing through the circle $x^2+y^2=25$, $y=4$.

Sol^n: Let (x, y, z) be some pt on the surface of the rt. circular cone. Let L be the st-line passing through (x, y, z) and $(0,0,0)$.

Let (x_0, y_0, z_0) be the pt. of intersection of L and the circle. Then $y_0 = 4$.

The eq^n of L is $\frac{x}{x_0} = \frac{y}{4} = \frac{z}{z_0} \Rightarrow x_0 = 4x/y$,
$z_0 = 4z/y$.

Putting these in the eq^n of the circle we get,

$$4^2 (x/y)^2 + 4^2 (z/y)^2 = 25 \Rightarrow 16(x^2 + z^2) = 25y^2 \text{ //.}$$

Convergence of a seq^n: Let $x_n = (x_{1,n}, x_{2,n}, x_{3,n}) \in \mathbb{R}^3$. We say that (x_n) is convergent if there exists $x_0 \in \mathbb{R}^3$ s.t. $\|x_n - x_0\| \to 0$ as $n \to \infty$. We then say $x_n \to x_0$.

• The seq^n x_n has three seq^{n}s attached to it $x_{1,n}, x_{2,n}, x_{3,n}$ whose properties help us understand the properties of x_n.

<u>Theorem</u>: (1) $x_n \to x_0$ in $\mathbb{R}^3$ iff the co-ordinates $x_{i,n} \to x_{i,0}$ for every $i = 1, 2, 3$ in $\mathbb{R}$.

(2) (x_n) is bounded iff each seq^n $(x_{i,n})$, $i = 1, 2, 3$ is bounded.

(3) Every bdd seq^n in $\mathbb{R}^3$ has a convergent subseq^n. (Bolzano - Weierstrass)

(4) Every convergent seq^n in $\mathbb{R}^3$ is bounded.