MA1012: Problem Sheet 5

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1. Using appropriate change of variables, show that
$$\int_0^1 \int_0^x (x-y)dydx = \int_0^1 \int_v^{2-v} \frac{v}{2} dudv.$$

2. Convert the integral $\int_0^1 \int_{x^2}^x dydx$ into one involving polar coordinates.

3. Evaluate the following integrals
$$\int_0^1 \int_0^{1-y} \sqrt{x+y}(y-2x)^2 dxdy \quad \text{and} \quad \int_0^{1/\sqrt{2}} \int_y^{\sqrt{1-y^2}} (x+y)dxdy.$$

4. Let D denote the solid bounded by the surfaces $y = x, y = x^2, z = x$ and $z = 0$. Evaluate the integral $\iiint_D ydxdydz$.

5. Evaluate the following integral
$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^4 xdzdydx.$$

6. Let D be the solid bounded above by the plane $z = 4$ and below by the cone $z = \sqrt{x^2+y^2}$. Evaluate the following integral
$$\iiint_D \sqrt{x^2+y^2+z^2} dxdydz.$$