MA1012: Problem Sheet 6

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1. Consider the paraboloid $z = x^2 + y^2 + 1$. Parametrize the surface by considering it as a graph, also parametrize the surface in the variables z and θ using cylindrical coordinates.

2. Parametrize the part of the sphere $x^2 + y^2 + z^2 = 16$, $-2 \leq z \leq 2$ using the spherical coordinates.

3. Let S denote the part of the plane $2x+5y+z = 10$ that lies inside the cylinder $x^2+y^2 = 9$. Find the area of S by considering S as a part of the graph $z = f(x, y)$, where
$$f(x, y) = 10 - 2x - 5y;$$
and, by considering S as a parametric surface
$$r(u, v) = (u \cos v, u \sin v, 10 - u(2 \cos v + 5 \sin v)) \quad 0 \leq u \leq 3 \quad \text{and} \quad 0 \leq v \leq 2\pi.$$

4. Let S be the part of the cylinder $y^2 + z^2 = 1$ that lies between the planes $x = 0$ and $x = 3$ in the first octant. Evaluate $\iint_S (z + 2xy) d\sigma$.

5. Let $f(x, y) = (xy, y^2)$, evaluate $\int_C f \cdot dR$, where C is the upper half of the unit circle traversing from $(-1, 0)$ to $(1, 0)$.

6. Let $F(x, y, z) = (2xy^2 + 3x^2, 2yx^2, 1)$, evaluate $\int_C F \cdot dR$ where C is the circular arc connecting $(0, 0, 0)$ and $(1, 1, 1)$.

7. Let $f(x, y, z) = (x^2, xy, 1)$, then show that there is no φ such that $\nabla\varphi = f$.