MA1012: Problem Sheet 7

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All plane curves C in this problem sheet are oriented counterclockwise.

1. Evaluate the line integral

$$\oint_C (x^2 \sin^2 x - y^3)dx + (y^2 \cos^2 y - y)dy,$$

   where C is the closed curve consisting of $x + y = 0, x^2 + y^2 = 25$ and $y = x$ and lying in the first and fourth quadrant.

2. Let a square R be enclosed by C and

$$\oint_C (xy^2 + x^3 \sin^3 x)dx + (x^2y + 2x)dy = 6.$$

   Find the area of the square.

3. Let C be a simple closed curve and $\alpha \in \mathbb{R}$, such that

$$\oint_C (\alpha e^x y + e^x)dx + (e^x + ye^y)dy = 0.$$

   Find the value of α.

4. Let D be the region enclosed by a simple closed smooth curve C. Show that

$$\text{Area of } D = \oint_C xdy = -\oint_C ydx.$$