MA1012: Problem Sheet 8

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1. Consider the surfaces $S_1 : x^2 + y^2 + z^2 = 4$ and $S_2 : x^2 + y^2 = 1$, let C be the curve of intersection of S_1 and S_2 and suppose C is oriented counterclockwise when viewed from above. Parametrize C.

2. Let C be the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$, suppose C is oriented counterclockwise when viewed from above, if $F(x, y, z) = (z, x, y)$, find $\oint_C F \cdot dR$; and if $\text{curl}F = \alpha\hat{k}$ for some $\alpha \in \mathbb{R}$ and $\oint_C F \cdot dr = 2\pi$, find α.

3. Let $F(x, y, z) = (z, x, y)$ and S be the part of the surface $2x^2 + 2y^2 + z^2 = 9$ that lies above the surface $z = \frac{1}{2}\sqrt{x^2 + y^2}$. Let C be the boundary of the surface which is oriented counterclockwise when viewed from above. Evaluate $\oint_C F \cdot dr$ using Stokes' theorem.

4. Let S be the upper hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$, evaluate $\iint_S (x^2 e^y - ye^y) d\sigma$.

5. Let D be the solid bounded by $z = 0$ and the paraboloid $z = 4 - x^2 - y^2$. Let S be the boundary of D. If $F(x, y, z) = (x^3 \cos(yz), y^3, x + \sin(xy))$, find $\iint_S F \cdot \hat{n} d\sigma$ where $\hat{n}$ is the unit outward normal to the surface S.

6. Let S be the sphere $x^2 + y^2 + (z - 1)^2 = 9$. Find the unit outward normal to the surface S and evaluate the surface integral

$$\iint_S (x^2 \sin y + y \cos^2 x + (z - 1)(y^2 - z \sin y)) d\sigma.$$

7. Let S be the sphere $x^2 + y^2 + z^2 = 1$, suppose for some $\alpha \in \mathbb{R}$ we have

$$\iint_S (zx + \alpha y^2 + xz) d\sigma = \frac{4\pi}{3}.$$

   Find the value of α.