Part A

1. The parametrization is
$$\left.\begin{aligned} x &= 2\sin\phi\cos\theta \\ y &= 2\sin\phi\sin\theta \\ z &= 2\cos\phi \end{aligned}\right\}, \quad \begin{aligned} &0 \le \phi \le \pi/4 \\ &0 \le \theta \le 2\pi . // \end{aligned}$$

2. If $\nabla f(x,y,z) = (3x^2, 2yz, y^2)$ for some f, then we have $f = x^3 + g(y,z)$ for some g.

   Since $f_y = g_y = 2yz \Rightarrow g = y^2 z + h(z)$ for some h.

   Since $f_z = y^2$ we get $f = x^3 + y^2 z$.

   The reqd. value is $f(1,1,1) - f(0,0,0) = 2 - 0 = 2 . //$

3. The reqd. volume is $\displaystyle\int_0^2\int_0^2 (16 - x^2 - 2y^2)\,dx\,dy = 48 . //$

4. The curl of a vector field F is denoted by curl F and is defined as curl F = $\nabla \times F$, where ∇ is the gradient field.

5. The given sphere S is $g(x,y,z) = 8$ where $g(x,y,z) = x^2+y^2+z^2$.
   The unit outward normal vector $\hat{n}$ of S is $\dfrac{\nabla g}{\|\nabla g\|} = \dfrac{1}{2\sqrt{2}}(x,y,z) . //$

Part B

6. In spherical coordinates, ϕ varies from 0 to $\pi/3$, $0 \le \theta \le 2\pi$, $a\sec\phi \le \rho \le 2a$. So, the reqd. integral is

$$\int_0^{\pi/3}\int_0^{2\pi}\int_{a\sec\phi}^{2a} \frac{\cos\phi}{\rho^2}\,|J(\rho,\theta,\phi)|\,d\rho\,d\theta\,d\phi$$

$$= 2\pi \int_0^{\pi/3} (2a\sin\phi\cos ec\,\phi - a\sin\phi)\,d\phi = \pi a/2 . //$$

②

7.(a) We have $\int_0^1 (\tan^{-1}\pi x - \tan^{-1}x)\,dx = \int_0^1 \left(\int_0^{\pi x} \frac{1}{1+y^2}\,dy \right) dx$

$$= \int_0^1 \int_{y/\pi}^{y} \frac{1}{1+y^2}\,dx\,dy + \int_1^{\pi} \int_{y/\pi}^{1} \frac{1}{1+y^2}\,dx\,dy. //$$

7.(b). The given solid lies above the region R, where R is in the 1st quadrant in $\mathbb{R}^2$ bounded by the circle $x^2+y^2=4$ and the lines $y=x$ and $y=0$.

So, the reqd. volume $= \iint_R \left(\sqrt{8-x^2-y^2} - \sqrt{x^2+y^2}\right) dx\,dy.$

Changing into polar co-ordinates we obtain that the reqd. vol^{m} is $\int_0^{\pi/4} \int_0^{2} \left(\sqrt{8-r^2} - r\right) r\,dr\,d\theta. = 4/3(\sqrt{2}-1)\pi. //$

8.(a) Let C be the curve and it is parametrized as,

$x(\theta) = \cos^3\theta,\ y(\theta) = \sin^3\theta,\ 0 \le \theta \le 2\pi.$

The reqd. area by Green's Theorem is,

$$A = \frac{1}{2}\oint_C x\,dy - y\,dx = \frac{3}{2}\int_0^{2\pi} \sin^2\theta\cos^2\theta\,d\theta$$

$$= \frac{3}{8}\int_0^{2\pi} \sin^2 2\theta\,d\theta = \frac{3}{8}\pi. //$$

8.(b). C is the boundary of the part of the surface $z = x+4$ that lies inside the cylinder $x^2+y^2=1$.

Curl $F = (0, 2z+e^z, 4)$.

By Stokes' theorem, $\oint_C F\cdot dR = \iint_D (0, 2z+e^z, 4)\cdot(-1,0,1)\,dx\,dy$

where D is the unit circle in the xy-plane.

So the reqd. value is $4\iint_D dx\,dy$. //